# ANSWER KEY

# FOR

# MULTIPLE-CHOICE & FREE-RESPONSE QUESTIONS IN PREPARATION FOR THE AP CALCULUS (BC) EXAMINATION

## (EIGHTH EDITION)

**By David Lederman**

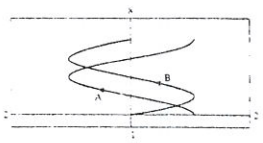
With the assistance of:

**Lin McMullin**

## SAMPLE EXAMINATION I

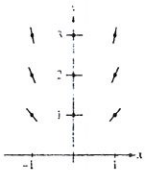
| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. C | 4. D | 7. E | 10. D | 13. E | 16. D | 19. B | 22. D | 25. A | 28. D | 31. B | 34. B | 37. D | 40. B | 43. B |
| 2. C | 5. A | 8. C | 11. D | 14. C | 17. A | 20. E | 23. E | 26. E | 29. B | 32. B | 35. D | 38. E | 41. D | 44. C |
| 3. B | 6. B | 9. C | 12. A | 15. A | 18. C | 21. B | 24. C | 27. E | 30. C | 33. C | 36. B | 39. E | 42. C | 45. E |

1. a. $y = -\frac{1}{2}\left(\sin\left(\frac{a}{100}\right)\right)x + \frac{a}{2}\sin\left(\frac{a}{100}\right) + 50\cos\left(\frac{a}{100}\right)$ b. $a = 45.935$ c. The top of the house cannot be seen by the person; see Solutions Manual.

2. a.  b. A $\langle -\sin t, 1\rangle$ B $\langle \cos t, 1\rangle$ c. A d. Both particles are moving upwards at the same rate.

3. a. $-8$ b. $-2$; speed is decreasing c. $t = 1$ d. farthest right at $t = 1$ and farthest left at $t = 4$

4. a. $\int_1^\infty \frac{1}{x^2}dx = 1$ b. $\int_1^\infty \left(1 - \frac{1}{x^2}\right)dx$; diverges c. $\int_1^\infty \left(\frac{1}{x^2}\right)^2 dx = \frac{1}{3}$

5. a.  b. $y = \dfrac{-1}{\frac{x^2}{2} + C}$ c. $y = \dfrac{-1}{\frac{x^2}{2} - 1} = \dfrac{-2}{x^2 - 2}$ d. $C \le 0$

6. $[-1, 1.5]$ b. $m = 2; G(0) = 7$ c. $G(3) = 7$

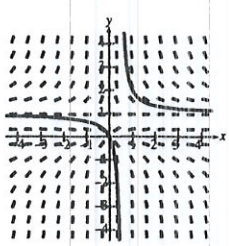
## SAMPLE EXAMINATION II

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. C | 4. C | 7. B | 10. D | 13. C | 16. E | 19. A | 22. C | 25. B | 28. D | 31. A | 34. E | 37. D | 40. C | 43. D |
| 2. B | 5. D | 8. A | 11. D | 14. D | 17. B | 20. B | 23. E | 26. D | 29. C | 32. B | 35. C | 38. B | 41. D | 44. D |
| 3. A | 6. D | 9. A | 12. E | 15. A | 18. E | 21. C | 24. C | 27. C | 30. A | 33. D | 36. E | 39. D | 42. C | 45. A |

1.a. 166.666 miles b. $-10$ miles/hour/hour c. Left Riemann sum ($n = 5$), $A \approx -72$ mph; Right Riemann sum ($n = 5$), $A \approx -52$ mph

2. a. 8:30 am b. 10:30 am c. \$18.26 d. \$9.47

3. a. 5.3 inches, less than true value b. 100 cubic inches per minute c. 11.3 inches; the integral represents the change in the depth of water in the tank, in inches, from $t = 0$ to $t = 15$ minutes. d. The approximation in part (c) is less than the true amount. Since $h''(t) > 0, h'(t)$ is increasing, and the left side values used in the approximation are all less than the true values.

4. a. see Solutions Manual b. $y = \dfrac{x}{x - \frac{1}{2}}; y'(0) = -2$ c. $x = \frac{1}{2}$ and $y = 1$ d. 

5. a. $\langle 2, -2t\rangle$ b. $y - (36 - t^2) = (-t)(x - 2t)$ or $y = -t(x - 2t) + (36 - t^2)$ c. $x = \dfrac{t^2 + 36}{t}$ d. 3

6. a. $y = -2ax + a^2 + 4$ b. $A = \int_0^a ((-2ax + a^2 + 4) - (4 - x^2))\,dx$ c. $A = \frac{1}{3}a^3$ d. $\dfrac{a^3}{12}$

## SAMPLE EXAMINATION III

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. B | 4. E | 7. D | 10. A | 13. E | 16. D | 19. C | 22. A | 25. D | 28. D | 31. E | 34. D | 37. E | 40. C | 43. D |
| 2. B | 5. B | 8. D | 11. B | 14. E | 17. A | 20. A | 23. C | 26. A | 29. E | 32. B | 35. C | 38. B | 41. A | 44. E |
| 3. C | 6. D | 9. B | 12. D | 15. D | 18. C | 21. E | 24. C | 27. A | 30. A | 33. D | 36. D | 39. E | 42. E | 45. C |

1. a. $a(t) = v'(t) = -\frac{1}{t^2}\cos\left(\frac{1}{t}\right)$ b. $t = \frac{3}{5\pi}$ or 0.1273 c. $-0.070$ or $-0.071$ d. left

2. a. 415.420 or 415.421 cubic meters. b. $-39.908$ or $-39.909$ cubic meters per hour. This means that the amount of sand in the bin is decreasing at the rate of 39.908 (or 39.909) cubic meters per hour when $t = 6$. c. The amount of sand, $A(t)$, in the bin is given by $A(t) = \int_0^t S(x)\,dx - \int_1^t R(x)\,dx$. The maximum occurs when $A'(t) = S(t) - R(t) = 0$ or when $S(t) = R(t)$. d. 0.00158 or 0.002 cubic meters.

3. a. $\sqrt{3} - \frac{\pi}{3}$ b. Volume $= \pi\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}\left((\sin(x))^2 - \left(\frac{1}{2}\right)^2\right)dx$ c. Volume $= \frac{\pi}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}\left(\frac{\sin(x) - \frac{1}{2}}{2}\right)^2 dx$

4. a. $-\tan t$ b. The slopes of the curved road and the two highways are the same at both A and B
c. $L = \int_0^{\frac{\pi}{2}}\sqrt{(-6\sin^2 t\cos t)^2 + (6\cos^2 t\sin t)^2}\,dt$

5. a. $-1 < x < 5$ b. see Solutions Manual; $f'(2) = -\frac{1}{3}$ c. $y - 1 = -\frac{1}{3}(x-2)$ or $y = 1 - \frac{1}{3}(x-2)$
d. Since the second derivative is positive at $x = 2$, the graph is concave up there and the tangent line lies below the graph of $f$.

6. $g(4.5) = 13.5; g'(4.5) = 0; g''(4.5) = -2$ b. $\frac{29}{8}$ c. $x = 5$ d. (4.5,13.5) and (9,13.25)

## SAMPLE EXAMINATION IV

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. B | 4. A | 7. E | 10. B | 13. D | 16. E | 19. D | 22. D | 25. E | 28. A | 31. E | 34. D | 37. C | 40. B | 43. E |
| 2. C | 5. E | 8. D | 11. C | 14. C | 17. E | 20. D | 23. D | 26. E | 29. A | 32. A | 35. C | 38. B | 41. E | 44. A |
| 3. C | 6. C | 9. C | 12. D | 15. A | 18. B | 21. E | 24. C | 27. D | 30. D | 33. C | 36. A | 39. E | 42. A | 45. E |

1. $\lim_{x\to-\infty} f(x) = 0; \lim_{x\to\infty} f(x) = 0$ b. $\frac{-3}{\sqrt{2\pi}}e^{-\frac{9}{2}}$ c. $x = 1$ and $x = -1$ d. 0.682 or 0.683

2. $k = 1.6\times10^7$ b. $L(\theta) = 10^4\cos^2\theta$ c. The strength is decreasing at $+1047.198$ lumens/sec.

3. a. $4 - \ln(5)$ b. $\pi\int_1^5\left(\left(2 - \frac{1}{x}\right)^2 - (2-1)^2\right)dx$ c. $\frac{30}{\ln 2} - 12$

4. a. $h(t) = \frac{1}{4}(8 - kt)^2$ b. $k = \frac{1}{8} = 0.125$ c. After 64 hours

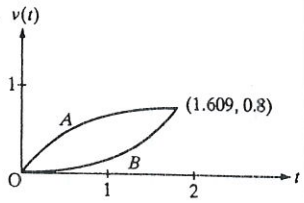
5. a. $(\sqrt{3},\sqrt{3})$ and $(-\sqrt{3},-\sqrt{3})$ b. $V(x) = 9x - x^3$ c. $6\sqrt{3}$

6. a. $y = 1$ b. $\cot x$ c. yes; see Solutions Manual d. $1 - \frac{1}{6}\left(x - \frac{\pi}{2}\right)^3$

## SAMPLE EXAMINATION V

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. E | 4. A | 7. B | 10. E | 13. B | 16. D | 19. E | 22. E | 25. C | 28. A | 31. B | 34. E | 37. B | 40. A | 43. B |
| 2. C | 5. C | 8. E | 11. B | 14. A | 17. D | 20. B | 23. D | 26. C | 29. E | 32. D | 35. D | 38. C | 41. C | 44. D |
| 3. C | 6. D | 9. B | 12. B | 15. C | 18. A | 21. D | 24. B | 27. B | 30. C | 33. E | 36. E | 39. C | 42. D | 45. A |

1. a. 40.172 b. 24.1644 c. $t = 1.4725079$ d. $y(3) = 20.0855$

2. a. 

b. A c. $\int_0^x ((1 - e^{-t}) - 0.2(e^t - 1))\,dt$ d. $t = 1.609$

3. a. $g(3) = -2.5; g'(3) = 0$ b. $x = 1, 3, 4.5$ c. $g''(x) = f'(x)$ d. $x = 1$ and $x = 4.5$ are relative maximums and $x = 3$ is a relative minimum by the Second Derivative Test.

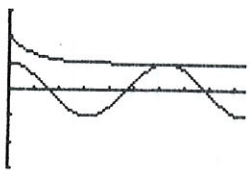
4. a. 60 b. $f(1)$ is the velocity of the train at the end of the first hour. The units are miles/hour. c. 1.35 miles

5. a. $y = 35 + 5\cos\theta$ b. $\theta = \frac{\pi}{30}t$ c. $\frac{dy}{dt} = -\frac{\pi}{6}\sin\theta$ d. 45 minutes after the hour

6. a. $T(t) = 1400 - k \cdot e^{-0.01t}$ b. $k = 1400$ c. safe

## SAMPLE EXAMINATION VI

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. E | 4. A | 7. C | 10. A | 13. C | 16. D | 19. D | 22. C | 25. B | 28. A | 31. D | 34. B | 37. B | 40. C | 43. E |
| 2. C | 5. E | 8. A | 11. E | 14. D | 17. A | 20. C | 23. A | 26. C | 29. D | 32. E | 35. A | 38. D | 41. E | 44. B |
| 3. A | 6. D | 9. D | 12. B | 15. B | 18. E | 21. E | 24. E | 27. D | 30. C | 33. B | 36. C | 39. B | 42. A | 45. C |

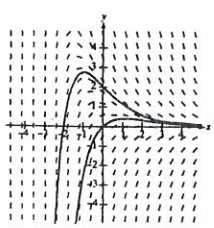
1. a. 
b. Since for all $x$, $1 + e^{-x} > 1$ and $\cos x \le 1$, the particles can never be at a point with the same $y$-coordinate, therefore they will never collide. c. 0.001 or 0.002 d. 2.04419 or 2.044

2. a. $R(t) = 10^6 e^{-0.092t}$ b. $t \approx 125$ seconds c. 7.534

3. a. $e + e^{-1} - 2$ b. $\pi\left(\frac{e^2 + e^{-2} - 2}{2}\right)$ c. $\int_0^1 x(e^x - e^{-x})\,dx$

4. a. $x(t) = 4\sin t$ and $y(t) = -\cos t + 1$ b. $\frac{1}{4}$ c. $t = \frac{\pi}{4}$

5. a. $1 + x^2 + \frac{2}{3}x^4$ b. $1 + \frac{1}{3}x^2 + \frac{2}{15}x^4$ c. 1 d. exact; see Solutions Manual

6. a. 
b. see Solutions Manual c. The maximum points are $(1 - C, e^{-(1-C)})$. They all lie on the graph of $f(x) = e^{-x}$