ONE SAMPLE/PAIRED T-INTERVAL

A local restaurant is trying to determine if they can raise their prices by looking at the incomes of the people in the community. They chose 7 people at random from the community and found their incomes, which are listed below.

30,000 34,000 42,000 50,000 55,000 60,000 60,000

Construct and interpret a 99% confidence interval for the average salary of all residents in the community

Know how to state your procedure and check the conditions for a one sample t-interval (13, 1b)

We are doing a one-sample t-interval → $CI = \bar{x} \pm t^* \cdot \frac{s}{\sqrt{n}}$

The conditions are

1) Random Sample→"they chose 7 people at random"
2) Normal Distribution→We can't use CLT, because 7 < 30. So, we would graph the distribution, and make sure that it did not deviate extremely from normal. Then we would claim, "appropriate to proceed." (Note: You actually have to create a graph. I did not do that in this example.)

Know how to calculate a confidence interval for a one sample t-interval, (13, 1b; 11b, 6d; 04, 6a; M2, 8)

$CI = 47285.71 \pm 3.707 \cdot \frac{12202.65}{\sqrt{7}} = (30186,64385)$

Know how to interpret the confidence interval for a one sample t-interval, (13, 1b; 04, 6a)

We are 99% confident that the average salary of people in this community is between $30186 and $64385

THE THREE STEPS LISTED ABOVE ARE WHAT SHOULD BE DONE ON EVERY CONFIDENCE INTERVAL TO GET FULL CREDIT

More on Confidence Intervals Part 1

What exactly is a confidence interval? How do I use it to argue for or against a claim (e.g. Based on your confidence interval is there evidence that boys are smarter than girls?)? (11b, 5a; 10, 3b;10b, 6d; 03, 6b)

Using the restaurant example on the previous page, is there evidence that the average salary is $65000?

No, because $65000 is not one of the choices in our confidence interval. It is unlikely that the average salary of the community is $65000

If I need to have a certain margin of error, how do I establish how many observations to make? (11b, 5b; 10, 3c; 08b, 3a; 05, 5b; 03b, 6b; M2, 26)—635, 671

Let's say that the people at our restaurant on the previous page didn't like that our confidence interval had such a large margin of error. Assuming that the population standard deviation is σ = $12000, how many people would then need to survey to reach a margin of error of at most $8000?

$m = z^* \cdot \frac{\sigma}{\sqrt{n}}$ --> $\mathbf{8000 \geq 2.576 \frac{12000}{\sqrt{n}}}$ --> **n ≥ 14.93**

They would need to survey at least 15 people**. Note: You always round up, even if it was 14.1, because 14 is not enough people to get the margin of error that you want**

Know how to interpret a confidence level (i.e. What does it mean to be 95% confident)? (10, 3a; 03b, 6a; M2, 37)

Using the restaurant situation we would say, "If many samples were taken from this community and many confidence intervals were created, 99% of those intervals would contain the true average salary of everyone in the community." **(Note: This interpretation applies to all confidence levels)**

Know how changing the sample size affects the margin of error, (M7, 30)

The more samples that you collect, the smaller your margin of error is. This makes sense, because if you take more observations, you should be getting a better estimate of the true average. **(Note: This applies to all confidence intervals)**

Know what happens to a confidence interval and the margin of error if you increase the confidence level, (M2, 13)

The more confident you are, the wider your confidence interval will become. In order to be extremely confident, you have to have a large margin of error. I am 100% confident that you will get a between a 1 and 5 on the AP exam. I am only 60% confident that you will get between a 3 and a 4. **(Note: This applies to all confidence intervals)**

Know when to do a t-test/interval instead of a z-test/interval, (M7, 25; M2, 33)

You do a t-test or inveral when you do not know the population standard deviation. In other words…

s → t σ → z Note: knowing σ is very unlikely. You will almost always do a t-interval

Know what a t-distribution is and how it is similar and different to a normal distribution, (M2, 18)

A t distribution is very similar to a normal distribution, except it has more area in the tails. So in a normal distribution going two standard deviations covers 95% of the data. In a t-distribution you would have to go more than 2 standard deviations to get 95% of the data.

TWO SAMPLE T-INTERVAL

In a certain large city, the mayor wants to know the difference in math scores between males and females. To measure this, a random sample of students were chosen to take the same test, and the results are shown below.

Males	Females
$\bar{x} = 35$	$\bar{x} = 40$
s = 5	s = 7
n = 40	n = 50

Create and interpret a 90% confidence interval for the average difference in math scores between male and female 12th graders in this city.

Know how to state your procedure and the check conditions for a two sample t-test/interval (11, 4; 12, 3b; 10, 5;09, 4a;08, 6a; 08b, 1b; 07b, 5; 06, 4; 05, 6a; 04b, 4a; 04b, 5c)—782

Two sample t-interval → $CI = (\bar{x}_m - \bar{x}_f) \pm t^* \sqrt{\frac{s_m^2}{n_m} + \frac{s_f^2}{n_m}}$

We need to verify that we have two independent samples and that both groups are normally distributed

1) Random → "a random sample of students"
2) Normal → Since both samples are larger than 30, the distribution of averages for both groups should be normal by Central Limit Theorem
3) Independents → No reason to believe that female scores should impact males scores

Know how to calculate a confidence interval comparing two sample means, (09, 4a; 06, 4; 05, 6a; 04b, 4a)—788

$CI = (35 - 40) \pm t^* \sqrt{\frac{5^2}{40} + \frac{7^2}{50}} = (\mathbf{-7.106}, \mathbf{-2.894})$ with df = 86.98

Know how to write a conclusion in context for a two sample/proportion t/z interval (09, 4a; 09, 5b; 06, 4; 06b, 2a; 05, 6a; 04b, 4a)

We are 90% confident that the average difference in math scores for male and female 12th graders (M – F) in this city is between -7.106 and -2.894.

THE THREE STEPS LISTED ABOVE ARE WHAT SHOULD BE DONE ON EVERY CONFIDENCE INTERVAL TO GET FULL CREDIT

Know how to use a confidence interval to determine if there is evidence that there is a difference between two populations, (09, 4b; 07, 1c; 06, 4; 06b, 2b, 05b, 4b)

For the example above, is there evidence that there is a difference between average male and female test scores?

Yes, because our interval does not include 0. 0 would imply that there is no difference. In fact, since both numbers are negative and we have a fairly high confidence in our interval (90%), we have evidence that females have a higher average than males.

Or… Yes, because our interval does not include 0. 0 would imply that there is no difference. In fact, since both numbers are negative and our confidence is 90%, we would reject at the $\alpha = 0.10$. Thus, we have evidence that females have a higher average than males.

Note: You should link your conclusion to 0 and to the confidence level or to the two sided alternative test

ONE SAMPLE/PAIRED T-TEST

A math teacher always makes two different tests, in order to reduce cheating. He is concerned that one of them is harder than the other. In order to test this, he grouped his students together by their current grade in the class. For example, he took the top student from each class and paired them together, the next best student from each class and paired them together, and so on. Then he had one person from each group take test A, and the other take test B. He determined who would get which test by flipping a coin. Below is a random sample of the test results.

	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7
Test A	50	43	61	70	80	36	51
Test B	60	45	61	65	85	42	55
Difference (Test B – A)	10	2	0	-5	5	6	4

Is there evidence that there is a difference between the two tests?

Know how to state your test and check the conditions for a one sample/paired t-test or confidence interval, (14, 5; 13, 1b; 09b, 5a;08b, 6c; 07, 4; 06b, 4; 05b, 4a; 05b, 6a; 04, 6a; 04b, 5b)

We are going to do a paired t-test → $t = \frac{\bar{x}_d - \mu_0}{\frac{s}{\sqrt{n}}}$ (Note: This is a 1 sample t-test, we just are looking at differences instead of just a single set of data. But, those difference create a single set of data)

The condition for a paired t-test are random samples and normal distribution

1) Random → Tests were assigned at random and these 7 groups were selected at random
2) Normal → If we created a boxplot of the 7 differences (10, 2, 0, -5, 5, 6, 4), since it doesn't vary extremely from normal we can say that it is appropriate to proceed.
    a. Note: You need to actually create a graph

Know how to write a pair of hypotheses for a one sample/paired t-test, including a definition of parameters, (14, 5; 09, 6a; 09b, 5a; 08b, 6c; 07, 4; 06, 6a; 06b, 4; 05B, 4a; 05b, 6a; 03, 1c; M7, 5)

$H_0: \mu_d = 0$ Let μ_d =The average difference in score between test B and test A

$H_a: \mu_d \neq 0$ **Note: You don't use the letter "d" unless it is a difference.**

Know how to calculate a test statistic for a one sample/paired t-test, find its degrees of freedom, and calculate its P-value, (14, 5; 09b, 5a;08b, 6c; 07, 4; 06b, 4; 05b, 4a; 05b, 6a; M2, 39)

$t = \frac{3.14 - 0}{\frac{4.78}{\sqrt{7}}} = 1.74$ df = 6 P = 0.1323

Know how to draw a conclusion in context based on a P-value in a one sample/paired t-test, (14, 5; 09b, 5a;08b, 6c; 07, 4; 06b, 4; 05b, 4a; 05b, 6a; M2, 24)

We cannot reject the null hypothesis in favor of the alternative at the 5%, because 0.1323 > 0.05. There is not significant test that this math teacher is making one test harder than the other.

THE FOUR STEPS LISTED ABOVE ARE WHAT SHOULD BE DONE FOR EVERY HYPOTHESIS TEST

More on Hypothesis Tests Part 1

Know when to do a paired t-test as opposed to a 2 sample t-test, (08b, 6c)

In the math test problem on the previous page, we did a paired t-test because the two numbers were not independent. It makes sense that if two students with similar class ranks are put together in a group then they should get similar grades. So, their scores will not be completely independent from each other.

Know what a Type 1 and Type 2 error, and be able to distinguish which one has worse consequences in the context of a problem if given a null and alternative hypothesis, (12, 5a; 11b, 4d; 09, 5c; 08b, 4b, 03, 2b)

In the math test problem on the previous page, this how the errors would be described.

Type 1: Thinking that there was a difference between the two tests, when in fact they are the same difficulty. A consequence would be that the teacher may have to use a different method when he creates his tests.

Type 2: Thinking that there was a difference between the two tests, when in fact one is more difficult than the other. A consequence would that this teacher would keep creating tests in this manner, and some students are getting a harder test than other students.

Note: These interpretations apply to all hypothesis tests

Know how to interpret a P-value in the context of a two sample/proportion problem. How is this different that just drawing a conclusion?, (09, 5a; 07, 5c; 07b, 6a)

Using the data from the math test problem from the previous page, this is how we would interpret the P-value.

If in fact there were no difference between the two tests (null hypothesis true), then there would be a 13.23% chance of randomly getting the numbers that we go, or numbers more extreme. Since that is a relatively high percentage, we did not reject the null hypothesis, because we believe that we could have randomly gotten this data even if the two tests were equally difficult.

Note: These interpretations apply to all P-values